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CAN GIANT PLANETS FORM BY DIRECT GRAVITATIONAL INSTABILITY?

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ABSTRACT

Gravitational instability has been invoked as a possible mechanism of giant planet formation in protoplanetary disks. Here we critically revise its viability by noting that for the direct production of giant planets it is not enough for protoplanetary disks to be gravitationally unstable. They must also be able to cool efficiently (on a timescale comparable to the local disk orbital period) to allow the formation of bound clumps by fragmentation. Combination of dynamical and thermal constraints puts very stringent lower limits on the surface density and temperature of disks capable of fragmenting into self-gravitating objects: for the gravitational instability to form giant planets at 10 AU gas temperature at this location must exceed 10^3 K for a minimum disk mass of 0.7 $M_\odot$ and minimum disk luminosity of 40 $L_\odot$. Although these requirements relax in more distant parts of the disk, masses of bound objects formed as a result of instability even at 100 AU are too large (~ 10 M_J) to explain characteristics of known extrasolar giant planets. Such protoplanetary disks (and planets formed in them) have very unusual observational properties and this severely constrains the possibility of giant planet formation by direct gravitational instability.

Subject headings: planets and satellites: formation — solar system: formation — planetary systems: protoplanetary disks

1. INTRODUCTION.

Recent discoveries of Jupiter-like planets around solar-type stars have rejuvenated the interest in the issue of the origin of giant planets. Core instability model (Perri & Cameron 1974; Harris 1978; Mizuno 1980) in which Jupiter-like planets acquire their massive gaseous atmospheres by unstable gas accretion onto the preexisting massive solid cores has been one of the most fruitful ideas in this field. For rather long time this avenue of planet formation did not seem compatible with the short observed lifetimes (10^6-10^7 yr) of protoplanetary disks because of the long time needed for the core accumulation. However, recent work by Rafikov (2003) and Goldreich et al. (2004) has found core formation time to be actually quite short, essentially removing the timescale issue from the core instability scenario. Nevertheless, gravitational instability (hereafter GI) in the protoplanetary disk (Cameron 1978; Boss 1998) has been put forward as an alternative mechanism of giant planet formation. In this model massive gaseous disk becomes gravitationally unstable and rapidly fragments into a number of self-gravitating bound structures, which further collapse to become giant planets. A number of recent hydrodynamical simulations (Boss 1998; Mayer et al. 2002) employing isothermal equation of state (EOS) have confirmed this general picture and provided ample support for the idea of planet formation by GI. The goal of this study is to constrain this avenue of planet formation by putting special emphasis on the conditions necessary for the actual fragmentation of protoplanetary disk into self-gravitating objects.

2. DYNAMICAL AND THERMAL CONSTRAINTS.

Gravitational instability in a Keplerian disk can operate only when the gas sound speed c_s, surface density Σ and local angular frequency Ω satisfy the condition

$$Q \equiv \frac{\Omega c_s}{\pi G \Sigma} < Q_0. \tag{1}$$

(Safronov 1960; Toomre 1964). Here Q is the so-called "Toomre Q"; sound speed is defined as $c_s \equiv (kT/\mu)^{1/2}$, where T is the disk midplane temperature, μ is the gas molecular weight, and k is a Boltzmann constant. Analytical arguments and results of numerical simulations suggest that $Q_0 \approx 1$. Equivalent way of formulating the condition for operation of GI is to require

$$\rho > \frac{\Omega^2}{\pi G Q_0} = 1.9 \times 10^{-7} \text{g cm}^{-3} a_{AU}^{-3} Q_0^{-1}, \tag{2}$$

where ρ is a midplane gas density and $a_{AU} \equiv a/(1\text{ AU})$ is the distance from the central star scaled by 1 AU.

Dynamical constraint (1) is a necessary condition for GI to set in. However, even when (1) is fulfilled, giant planet formation becomes possible only if the disk can actually *fragment* into bound self-gravitating objects. Recent studies (Gammie 2001; Rice et al. 2003) have demonstrated that fragmentation is possible only provided that the cooling time of the disk t_{cool} satisfies

$$\Omega t_{cool} < \xi, \tag{3}$$

where ξ is a parameter of the order of unity; numerical simulations (Gammie 2001; Rice et al. 2003) suggest that $\xi \approx 3$.

Cooling time of the disk can be estimated as the ratio of its thermal energy to the escaping radiative flux:

$$t_{cool} \approx \frac{\Sigma c_s^2}{\gamma - 1} \frac{f(\tau)}{\sigma T^4}, \quad f(\tau) = \tau + \frac{1}{\tau}, \tag{4}$$

where σ is the Stephan-Boltzmann constant, T is the midplane disk temperature, $\tau \approx \kappa\Sigma/2$ is the disk optical depth (κ is the opacity), and γ is the adiabatic index of

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gas. Function $f(\tau)$ describes the efficiency of disk cooling and its specific form in (4) corresponds to the case when cooling is radiative. When the disk is optically thick, $\tau \gg 1$, radiation has to leak out through the large optical depth of the disk material; this lowers the effective temperature at the disk photosphere by a factor of $\tau^{1/4}$ compared to the midplane temperature T and makes t_{cool} very long [first term in the definition of $f(\tau)$]. In the optically thin case, $\tau \ll 1$, according to the Kirchhoff's law (Rybicki & Lightman 1979) disk emissivity is low and t_{cool} is again very large [second term in the definition of $f(\tau)$]. Thus, disk cools most effectively when $\tau \approx 1$. What is most important for our further discussion is that the value of $f(\tau)$ is above unity for any τ and any cooling mechanism because effective temperature of the disk cannot exceed its midplane temperature.

Dynamical constraint (1) limits the value of the sound speed from above. At the same time, expressing the temperature in (4) through c_s we find that (3) sets a lower limit on the gas sound speed. Combination of these conditions leads to the following constraint on c_s necessary for the giant planet formation by GI:

$$\left[\Sigma \frac{f(\tau)}{\zeta}\frac{\Omega}{\sigma}\left(\frac{k}{\mu}\right)^4\right]^{1/6} < c_s < \pi Q_0 \frac{G\Sigma}{\Omega}, \qquad (5)$$

where $\zeta \equiv \xi(\gamma-1) \approx 1$. Only when (5) is fulfilled can the disk be gravitationally unstable and cooling be fast enough for fragmentation to allow the formation of bound gaseous clumps, which later collapse to become giant planets. Similar argument has been advanced by Levin (2003) in application to self-gravitating disks around AGNs.

At a specific location in the protoplanetary disk the condition (5) can be satisfied only provided that the gas surface density obeys

$$\Sigma > \Sigma_{min} \equiv \Sigma_{inf}[f(\tau)]^{1/5} \quad \text{where} \qquad (6)$$

$$\Sigma_{inf} \equiv \Omega^{7/5}(\pi G Q_0)^{-6/5}\left[\frac{1}{\zeta\sigma}\left(\frac{k}{\mu}\right)^4\right]^{1/5}$$

$$\approx 6.6\times 10^5 \text{g cm}^{-2} a_{AU}^{-21/10}\left(Q_0^6\tilde{\mu}^4\zeta\right)^{-1/5}\left(\frac{M_\star}{M_\odot}\right)^{7/10} \qquad (7)$$

Here $\tilde{\mu} \equiv \mu/m_H$ is the mean molecular weight relative to the atomic hydrogen mass m_H and $M_\star$ is the mass of the central star ($M_\odot$ is the Solar mass). For molecular gas of solar composition $\tilde{\mu} \approx 2.3$ and $\Sigma_{min} \approx 3.4\times 10^5$ g cm^{-2} at 1 AU [for $f(\tau)/(Q_0^6\tilde{\mu}^4\zeta) = 1$], while if hydrogen is atomic $\tilde{\mu} \approx 1.2$ and $\Sigma_{min} \approx 5.6\times 10^5$ g cm^{-2}. In both cases the disk surface density at 1 AU turns out to exceed that of the minimum mass Solar nebula (Hayashi 1981) by more than 10^2, implying that a very massive protoplanetary disk is necessary to sustain giant planet formation by GI.

According to (5), whenever (7) is fulfilled, the sound speed in the disk is also bounded from below:

$$c_s > c_{s,min} \equiv \Omega^{2/5}\left[\frac{f(\tau)}{\zeta\pi Q_0 G\sigma}\left(\frac{k}{\mu}\right)^4\right]^{1/5}$$

$$\approx 6.9 \text{ km s}^{-1} a_{AU}^{-3/5}\left[\frac{f(\tau)}{Q_0\tilde{\mu}^4\zeta}\right]^{1/5}\left(\frac{M_\star}{M_\odot}\right)^{1/5}. \qquad (8)$$

Because of that midplane temperature has to satisfy

$$T > T_{min} \equiv T_{inf}[f(\tau)]^{2/5} \quad \text{where} \qquad (9)$$

$$T_{inf} \equiv \Omega^{4/5}\left(\zeta\pi Q_0 G\sigma\right)^{-2/5}\left(\frac{k}{\mu}\right)^{3/5}$$

$$\approx 5800 \text{ K } a_{AU}^{-6/5}\tilde{\mu}^{-3/5}\left(Q_0\zeta\right)^{-2/5}\left(\frac{M_\star}{M_\odot}\right)^{2/5}. \qquad (10)$$

In the form given by (6)-(10) constraints on the disk surface density and temperature still depend on the behavior of opacity. However, bearing in mind that $f(\tau) > 1$ for any τ, one immediately sees that values of Σ_{inf} and T_{inf} [1] represent *absolute opacity-independent lower limits* on the disk surface density and temperature. Thus, we conclude that for the gravitationally unstable disk to be able to fragment (which is necessary for giant planets formation) disk has to satisfy *at least* $\Sigma > \Sigma_{inf}$ and $T > T_{inf}$. These lower limits are extremely robust since they do not depend on a specific mechanism of energy losses from the disk — by radiation or by convection. In practice, taking into account even the most basic properties of the energy transfer within the disk one can formulate much more stringent constraints, as we demonstrate below.

3. APPLICATION TO PROTOPLANETARY DISKS.

Protoplanetary disk luminosity L_d and mass M_d are the characteristics which can be directly observed (or constrained using observations). According to (6), mass of a protoplanetary disk which is capable of planet formation in the range of semimajor axes from a_{in} to a_{out} has to satisfy

$$M_d > 4.7 M_\odot\left(a_{in,AU}^{-1/10} - a_{out,AU}^{-1/10}\right)\tilde{\mu}^{-4/5}\left(\frac{M_\star}{M_\odot}\right)^{7/10} \qquad (11)$$

This and all subsequent numerical estimates assume $Q_0 = 1$ and $\zeta = 1$. Note a rather weak dependence of M_d on the disk dimensions. For the inner disk cutoff at $a_{in} = 0.1$ AU and outer disk cutoff at $a_{out} = 100$ AU one finds that $M_d \gtrsim 3$ $M_\odot$ (for $\tilde{\mu} = 1$). This estimate assumes maximum cooling efficiency throughout the whole disk, i.e $f(\tau) = 1$, but even such lowest possible limit on M_d well exceeds the mass of the central star.

This constraint on the disk mass can be somewhat relaxed if disk is producing planets only locally. Indeed, the typical scale length of the fastest-growing perturbation in the marginally gravitationally unstable disk ($Q \approx 1$) is $\lambda \approx 2\pi h$, where $h \equiv c_s/\Omega$ is the vertical disk scaleheight. Thus, to form planets by GI it is, in principle, sufficient that only an annulus of the disk with the radial width of order λ has surface density and temperature exceeding Σ_{min} and T_{min}. Using (8) we can estimate

$$\frac{h}{a} > \frac{1}{\Omega^{3/5} a}\left[\frac{f(\tau)}{\zeta\pi Q_0 G\sigma}\left(\frac{k}{\mu}\right)^4\right]^{1/5}$$

$$\approx 0.23\, a_{AU}^{-1/10}\left[\frac{f(\tau)}{\tilde{\mu}^4}\right]^{1/5}\left(\frac{M_\star}{M_\odot}\right)^{-3/10}. \qquad (12)$$

Because of the high midplane temperature (necessary to ensure efficient cooling) disk is not very thin geometrically; as a result, GI is likely to have global character.

[1] From *infimum* — the greatest lower bound of a set.

Mass of such a "minimum planet-forming annulus" centered on a and having width λ is constrained by

$$M_a \gtrsim 0.8 M_\odot a_{AU}^{-1/5} \left[f(\tau)\right]^{2/5} \tilde{\mu}^{-8/5} \left(\frac{M_\star}{M_\odot}\right)^{2/5}. \quad (13)$$

Clearly, this lower limit on M_a depends only very weakly on the semimajor axis of annulus. It also amounts to a sizable fraction of $M_\star$ [even for molecular gas with $\tilde{\mu} = 2.3$ and $f(\tau) = 1$ one finds $M_a \gtrsim 0.2$ M$_\odot$ at 1 AU]. At the same time, typical masses of protoplanetary disks inferred from mm and infrared observations vary between 10^{-3} M$_\odot$ and 0.1 M$_\odot$ (Kitamura et al. 2002), but these masses of gas are typically extended over ~ 100 AU in disk radius (and not within just a narrow annulus). Besides, in the real nebula such annulus can easily be either very optically thick or very optically thin, which translates into large value of $f(\tau)$, additionally increasing the lower limit on M_a (see below).

Energy flux emitted from both sides by a unit surface area of the disk is given by

$$\frac{dL_d}{dS} = 2\frac{\sigma T^4}{f(\tau)} > 2\Omega^{16/5} \left[\frac{f(\tau)}{\sigma}\right]^{3/5} \left[\frac{(k/\mu)^{3/2}}{\pi G}\right]^{8/5} \quad (14)$$

where we made use of (10). Since $dL_d/dS \propto a^{-24/5}$ most of the energy is radiated by the innermost part of the gravitationally unstable disk. Total luminosity of a disk with an inner cutoff at a is

$$L_d = 2 \times 2\pi \int_a \frac{dL_d}{dS} a da \approx \frac{10\pi}{7} a^2 \frac{dL_d}{dS}(a) >$$

$$1.6 \times 10^4 L_\odot \, a_{AU}^{-14/5} \left[\frac{f(\tau)}{\tilde{\mu}^4}\right]^{3/5} \left(\frac{M_\star}{M_\odot}\right)^{8/5}. \quad (15)$$

This estimate of luminosity holds even if only an annulus of gas is considered instead of the full disk, which is a direct consequence of the very steep dependence of dL_d/dS on the distance from the central star.

We now consider what these limits imply for the disk properties at different locations in the protoplanetary nebula.

3.1. *Limits on the disk properties at* 1 *AU.*

First we consider the possibility of giant planet formation by GI in the region of terrestrial planets, at $a = 1$ AU. Constraints obtained in previous sections imply that planetary genesis at this location requires rather extreme disk properties: temperature has to exceed at least $T_{inf} = 5.2 \times 10^3$ K (this estimate is based using $\tilde{\mu} = 1.2$ in [10] since gas cannot be molecular at such temperature), surface density must be above $\Sigma_{inf} = 5.7 \times 10^5$ g cm^{-2}, and disk luminosity has to exceed 10^4 L$_\odot$ (all assuming $\zeta = 1$ and $Q_0 = 1$). Depending on whether an extended disk or just a minimum size annulus is considered, minimum mass varies from $M_d \approx 1.5$ M$_\odot$ to $M_a \approx 0.6$ M$_\odot$. Apparently, even these least radical requirements are in complete disagreement with the observed properties of protoplanetary disks.

The argument can be significantly sharpened by noticing that the disk with such high surface density has to be optically very thick, meaning that Σ_{min} and T_{min} provide much more stringent limits on the disk properties than Σ_{inf} and T_{inf}. Using opacity dependence $\kappa(\rho, T)$ on the temperature T and gas density ρ (given by [2]) from Bell & Lin (1994) we can substitute $\tau = \kappa(\rho, T)\Sigma_{min}/2$ into (6) and (9) and solve the resulting system for Σ_{min} and T_{min}. Performing this procedure one finds that at 1 AU disk has to be extremely hot so that opacity is due to electron scattering. Disk temperature has to exceed 10^6 K, but this is impossible, because such disk would not be bound to the central star and its radiation pressure would far dominate over the gas pressure. This firmly rules out the possibility of giant planet formation by GI within several AU from the central star.

3.2. *Limits on the disk properties at* 10 *AU.*

At 10 AU, in the region of giant planets, temperature will probably be low enough for the gas to be molecular. Using $\tilde{\mu} = 2.3$ we find $\Sigma_{inf} = 2.7 \times 10^3$ g cm^{-3} and $T_{inf} = 220$ K. Disk luminosity has to exceed only 3.4 L$_\odot$, and the full disk and minimum mass annulus have to contain at least $M_d = 0.4$ M$_\odot$ (for $a_{out} = 100$ AU) and $M_a = 0.13$ M$_\odot$ correspondingly. These limits are more reasonable than at 1 AU, although mass is still high and disk is too hot compared to the observed systems, which typically have $M_d < 0.1$ M$_\odot$ and $T \lesssim 10^2$ K at 10 AU (e.g. Kitamura et al. 2002).

However, using again opacities from Bell & Lin (1994) and ρ as given by (2) we find that real disks (i.e. cooling not at the maximum efficiency) should be considerably more extreme. There are several possible solutions for T_{min} and Σ_{min} at 10 AU corresponding to different opacity regimes. The least extreme one is a "cold" solution with $\tau \approx 60$, $T_{min} \approx 1100$ K, $\Sigma_{min} \approx 6 \times 10^3$ g cm^{-2}, $L_d \gtrsim 40$ L$_\odot$, $M_a \gtrsim 0.7$ M$_\odot$ which corresponds to molecular gas at the temperature of grain evaporation (other solutions have $T_{min} \gtrsim 7 \times 10^3$ K). This again yields a minimum disk which is too massive and hot to satisfy current observational constraints. Thus, it is extremely unlikely that GI can allow disk fragmentation and subsequent giant planet formation even at 10 AU.

3.3. *Limits on the disk properties at* 100 *AU.*

We also look at the possibility of giant planet formation by GI in the distant regions of protoplanetary nebula, at $a = 100$ AU. For molecular disk at this location we find $\Sigma_{inf} = 20$ g cm^{-3}, $T_{inf} = 14$ K, $L_d \gtrsim 5 \times 10^{-3}$ L$_\odot$, and $M_a \gtrsim 0.08$ M$_\odot$. These properties change only a little when inefficiency of disk cooling is properly accounted for: using $\kappa \approx 0.1(T/10\text{ K})$ cm^2 g^{-1} in agreement with observations of protoplanetary nebulae (Beckwith et al. 1990; Kitamura et al. 2002) we find that to be able to form giant planets by GI disk has possess at least $\tau \approx 2$ (marginally optically thick), $T_{min} \approx 20$ K, $\Sigma_{min} \approx 25$ g cm^{-2}, $L_d \gtrsim 10^{-2}$ L$_\odot$, and $M_a \gtrsim 0.1$ M$_\odot$ at 100 AU. Although M_a is very near the upper end of the observed distribution of protoplanetary disk masses, these parameters seem to be acceptable from the observational point of view. There are however additional reasons to doubt the possibility of planet formation by GI even at 100 AU, which we discuss next.

3.4. *Fragment masses.*

Another important observational constraint on the planet formation by GI comes from comparing the observed masses of extrasolar giant planets with the typical masses of fragments into which disk breaks up when

$t_{cool} < \xi\Omega^{-1}$. As we mentioned earlier, the lengthscale of the most unstable mode is $\lambda \approx 2\pi h$, which results in the minimum fragment mass of

$$M_f \approx \Sigma_{min}\lambda^2 \approx 0.15 \ \mathrm{M}_\odot a_{AU}^{-3/10} \left[\frac{f(\tau)}{\tilde{\mu}^4}\right]^{3/5} \left(\frac{M_\star}{M_\odot}\right)^{1/10} \quad (16)$$

At 100 AU molecular disk meeting the requirements outlined in §3 3.3 would fragment into self-gravitating clumps with the mass of roughly 5 $\mathrm{M}_J[f(\tau)]^{3/5}$, where M_J is the Jupiter's mass; at smaller semimajor axes M_f would be higher. Even for $f(\tau) = 1$ this mass is larger than masses of most extrasolar giant planets detected to date (Marcy et al. 2003). More realistic cooling efficiency corresponding to the optical depth of $\tau \approx 2$ at 100 AU leads to $M_f \approx 9$ M_J, landing minimum fragment mass not too far from the brown dwarf regime.

It is possible that such clumps would be able to further fragment into smaller objects but we view this outcome as rather unlikely. Indeed, as the fragment contracts its optical depth increases and cooling time in this nonlinear regime becomes larger than the dynamical time of collapsing fragment, which disfavors subsequent fragmentation (Goodman & Tan 2004). It remains to be seen how the centrifugal support of the collapsing clump can change this conclusion.

4. DISCUSSION.

Novel analytical constraints presented in §2 and 3, when confronted with observations of protoplanetary disks, severely undermine the possibility of giant planet formation by GI. In particular, we have demonstrated that disks capable of producing giant planets by GI at a distance of several AU from the central star cannot exist simply on dynamical grounds — to cool efficiently they must be too hot to be bound to the central object. This essentially rules out a possibility of in situ formation of close-in extrasolar giant planets ("hot Jupiters") by GI. Rafikov (2004) have previously presented arguments against in situ formation of "hot Jupiters" via the core instability. Then the most natural way to explain the existence of close-in giant planets is to accept that they have formed elsewhere under more favorable conditions and then migrated to their current locations.

Planet formation by GI is also extremely unlikely within several tens of AU, as results of §3 3.2 demonstrate: disks with required properties must be so hot ($T \gtrsim 10^3$ K), luminous (several tens of $L_\odot$), and massive ($\sim M_\odot$) that they would clearly stand out in a sample of observed protoplanetary nebulae. Beyond about 100 AU minimum disk properties allowing planets to form by GI become roughly acceptable from the observational point of view, although disk masses still reside at the very upper end of the observed distribution of protoplanetary nebulae masses. However, it is still very unlikely that the extrasolar giant planets that we see now could have been produced by GI even at several hundred AU from the central star. The problem is not only in a potential difficulty of migrating such planets from beyond 100 AU all the way in to several AU, but also in producing planets with the right mass. As our estimate (16) of the minimum mass of unstable disk fragments demonstrates, bound objects produced by GI even at 100 AU are too massive (~ 10 M_J) to explain the observed mass distribution of extrasolar planets.

Our study emphasizes the importance of the proper treatment of disk thermodynamics (especially its cooling) for studying the possibility of Jupiter-like planet formation. By now virtually all simulations which were able to demonstrate disk fragmentation and collapse of resulting dense objects in gravitationally unstable disks used isothermal EOS (e.g. Mayer et al. 2002). However, use of this EOS is equivalent to setting the disk cooling time to zero which artificially relaxes the requirements for the planet production process and is misleading. Not surprisingly, more realistic calculations following thermodynamics in greater detail typically do not exhibit fragmentation of gravitationally unstable disks which are not capable of cooling efficiently (Pickett et al. 1998; Gammie 2001; Rice et al. 2003). Thus, simulations employing isothermal EOS should not be trusted too much when planet formation in real protoplanetary disks is concerned.

Future infrared and mm observation will show whether protoplanetary disks with extreme properties satisfying the constraints necessary for giant planet formation by GI really exist.

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